Quantum Gravity

The Probability Wave Dispersion Interpretation of Relativity

Todd Desiato

Physics

8088 Flanders Drive

San Diego, CA, 92126

Release: 3

Revision: B

March 14, 1999

Abstract

The possibility that gravitation results from the dispersion of probability waves is explored. The wave functions chosen are solutions to the Dirac equation in a Minkowski space-time. It is hypothesized that the shape of the space-time manifold is determined by the polarizability of the wave functions at each point on the manifold. An analogy is made to an electronic circuit representing the polarizability of the manifold by the equivalent inductance and capacitance value at each point. The metric components are derived from this hypothesis and are applied to the energy momentum four-vector of the wave solution. The probability wave dispersion relationship is found, and its relationship to real and pseudo forces is explored in terms of how this brings about the properties of inertia and space-time curvature. The Christoffel field from which the field equation can be calculated is derived, and the Schwarzschild solution of the vacuum equation of General Relativity is shown to fit the model. A dispersion diagram is also derived and is shown to be equivalent to a Minkowski space-time diagram. As a result of this hypothesis and the dispersion interpretation of Relativity, General Relativity appears to be derivable from Quantum Electrodynamics.

Introduction

The intent of this paper is to illustrate a new interpretation of how gravitation at the quantum scale can be modeled as an electronic circuit. This is done in the hopes that it will benefit engineers who wish to investigate the possibility of designing electro-gravitic devices. In so doing the attempt is made where possible to translate the fluent geometrical language of General Relativity into the less rigorous, but more intuitive language of electrical engineering. A basic understanding of Quantum Mechanics, vectors and the index notation of General Relativity are required. As well as a rudimentary knowledge of electronic circuit analysis.

The equivalent circuit for the transmission of probability waves through space-time is shown to be analogous to the transmission of waves through wave guides in quantum optics, or along transmission lines in electrodynamics. With this interpretation it is shown that the relationships between gravitation and electrodynamics compliment each other in the same fashion that the components of an electronic circuit compliment the sources of electrical energy.

What is presented here shows how the gravitational field can be modeled by variable component values in the equivalent circuit. In a linear circuit the component values do not depend on the strength of the sources. In reality however components such as inductors and capacitors, along with some integrated circuits do have non-linear properties that can also be added to this model in the future.

The Propagator

The *propagator* is defined as the probability amplitude that a particle is observed at the space-time point $B(x_2, t_2)$, when it is known that the particle is at the point $A(x_1, t_1)$. The propagator $K(B \mid A)$, is the integral over all possible paths from the point A to the point B. It is a very difficult integral since there are an infinite number of paths between any two points. Let the function θ_l be the probability amplitude that the particle actually takes the path l between A and B. It is referred to as a *phase factor*. If the point A is held fixed, then the propagator is only a function of the location B, and can simply be written as the probability wave solution of the Dirac equation for a free electron.

The wave equation for a free electron is,

$$\psi_{\pm}(x, p) = \varsigma^{\ell}(p) \exp\left[\frac{i}{\hbar} p^{\alpha} g_{\alpha\beta} x^{\beta}\right] \tag{1}$$

where p^{α} is the momentum four-vector, x^{β} are the coordinate functions and $g_{\alpha\beta}$ is the space-time metric tensor. The electron traveling along an arbitrary path l connecting the points $A(x_1, t_1), B(x_2, t_2)$ adds to the propagator a phase factor,

$$\begin{aligned} \theta_l{}^0(B \mid A) &= \exp\left[\frac{i}{\hbar} \int_{t_1}^{t_2} p^0 g_{00} \partial x^0\right] \\ &= \exp\left[\frac{i}{\hbar} \int_{t_1}^{t_2} E \, dt\right] \end{aligned} \tag{2}$$

This is simply the contribution of *time evolution* to the propagator. [1]

Probability Wave Dispersion

The function $S_0(l)$ is called the *action* of the path, and to each path we define an action,

$$S_0(l) = \int_{t_l} E \, dt \tag{3}$$

It is weighted against Planck's constant, which also has units of action per radian.

In a Schwarzschild space-time the metric component g_{00} is,

$$g_{00} = 1 - \frac{2GM}{r} = 1 - g(r, M) \tag{4}$$

and the action can be written as,

$$\begin{aligned} S_0(l) &= \int_{t_1}^{t_2} p^0 (1 - g(r, M)) \partial x^0 \\ S_0(l) &= S_1(l) - S_g(l) \end{aligned} \tag{5}$$

It is immediately evident how the curved Schwarzschild metric affects the propagator by adding a phase shift to the time evolution,

$$\theta_l{}^0 (B \mid A) = \exp\left[\frac{i}{\hbar} S_1(l)\right] \exp\left[\frac{-i}{\hbar} S_g(l)\right] \tag{6}$$

This is Probability Wave Dispersion! It is the result of the gravitational field acting on the wave function of the free electron.

Interaction with the Electromagnetic Field

The presence of an electromagnetic field adds the interaction term, (superscript I),

$$S_0{}^I(l) = q \int_{t_1}^{t_2} A^\alpha g_{\alpha\beta} \partial x^\beta \tag{7}$$

which is the action of the electromagnetic *Lorentz force*. This is the quantum mechanical Bohm-Aharonov Effect, where q is the charge and A^α is the electromagnetic gauge potential. In the absence of a magnetic field this becomes,

$$\begin{aligned} S_0{}^I(l) &= q \int_{t_1}^{t_2} A^0 g_{00} \partial x^0 \\ S_0{}^I(l) &= q \int_{t_1}^{t_2} A^0 (1 - g(r, M)) \partial x^0 \\ S_0{}^I(l) &= S_1{}^I(l) - S_g{}^I(l) \end{aligned} \tag{8}$$

The affect on the probability wave function is once again the addition of an interaction phase factor.

$$\theta_l{}^I (B|A) = \exp\left[\frac{i}{\hbar} S_1{}^I(l)\right] \exp\left[\frac{-i}{\hbar} S_g{}^I(l)\right] \tag{9}$$

The phase shift added to the propagator in a static *electro-gravitic* field is then,

$$\theta_l^{\,0}(B|A)\theta_l^{\,I}(B|A) = \exp\left[\frac{i}{\hbar}S_1(l)\right]\exp\left[\frac{-i}{\hbar}S_g(l)\right]\exp\left[\frac{i}{\hbar}S_1^{\,I}(l)\right]\exp\left[\frac{-i}{\hbar}S_g^{\,I}(l)\right] \qquad (10)$$

This shows that both the gravitational field and electromagnetic field act upon the propagator in the same manner. Both induce a phase shift which causes the wave function to disperse in the direction which minimizes the action, thereby decreasing the relative energy of the wave. The difference is that the gravitational field contribution is the result of the mass/energy, and the electromagnetic field contribution is a result of the electromagnetic energy. Both fields are representative of the local environment of a test particle.

The Capacitance Gradient

Starting with the Coulomb gauge potential of the electric field for a single electron of charge $-e = q$,

$$\phi(r) = \frac{-e}{4\pi\varepsilon_0 r} = \frac{q}{C(r)} \qquad (11)$$

The potential of an electric field is its voltage. The energy of the electric field is stored in a capacitance $C(r)$. A capacitor is simply described as a thin tube of electric flux in which the energy of the field is stored. Parallel tubes of flux effectively form parallel capacitors called *guard capacitors*, so there is no leakage from the tube. No conductors are required in order to have capacitance, as the electromagnetic field does not require a medium in which to store energy.

The electric field is the gradient of the potential,

$$\begin{aligned}\vec{\mathbf{E}}(r) &= -\vec{\nabla}\phi(r)\\ \vec{\mathbf{E}}(r) &= -q\,\vec{\nabla}\frac{1}{C(r)}\end{aligned} \qquad (12)$$

A change in the radial coordinate position is equivalent to a change in the capacitance. The reciprocal of capacitance is the *elastance* $D(r)$. If the charge is held constant then the gradient of the capacitance is a constant vector,

$$\begin{aligned}C(r) &= 4\pi\varepsilon_0 r\\ \vec{\nabla}C(r) &= 4\pi\varepsilon_0\hat{\mathbf{r}}\ \ \mathbf{Farads/m}\end{aligned} \qquad (13)$$

This can be written more compactly as a four-vector. Introducing the Minkowski metric tensor,

$$\eta_{\alpha\beta} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \tag{14}$$

In this paper lower case Greek alphabet characters as superscripts represent contravariant four-vectors, and as subscripts represent covariant four-vectors. Indices are raised and lowered using the metric tensor. Three-vectors are shown with an arrow to indicate their three dimensional vector nature.

As a four-vector the capacitance gradient at the space-time point P is,

$$\partial_\alpha C(P) = \varepsilon_\alpha \ \textbf{Farads / m} \tag{15}$$

Capacitance is thus shown to be an *effective potential* who's gradient leads to a constant ratio of capacitance per unit length, represented by a permittivity vector ε_α. A function of the constant of permittivity and a geometrical variable.

The Inductance Gradient

Now consider a region where there is no static charge, but there is electric current flowing such that the magnetic vector potential is not zero. The magnetic flux is the integral of the flux density through the surface area S bound by the closed curve ℓ. By Stokes theorem it is also the integral of the magnetic vector potential around the closed curve ℓ.

$$\Phi(r) = \int_S \vec{\mathbf{B}} \bullet \hat{\mathbf{n}}\, dA = \int_S \left(\vec{\nabla} \times \vec{\mathbf{A}}\right) \bullet \hat{\mathbf{n}}\, dA = \oint_\ell \vec{\mathbf{A}} \bullet d\vec{\mathbf{r}} \quad \textbf{Volt - sec} \tag{16}$$

The magnetic flux is valued as the inductance times the current.

$$\Phi(r) = L(r) \cdot I \tag{17}$$

An inductor is defined as a tube of magnetic flux with no conductors required. The energy of the magnetic field is stored in the inductance. If for example $\vec{\mathbf{A}}$ is everywhere constant and parallel to an infinitesimal length $d\vec{\mathbf{r}}$ around a small closed circle, or $\vec{\mathbf{B}}$ is everywhere orthogonal to the bound surface S, then the inductance can be calculated as simple integrals of the loop,

$$\begin{aligned} L(r) &= \frac{\pi r^2 \left|\vec{\mathbf{B}}\right|}{I} = \frac{2\pi r \left|\vec{\mathbf{A}}\right|}{I} \cdot \quad \textbf{Volt - sec / Ampere} \\ \left|\vec{\mathbf{B}}\right| &= \frac{\mu_0 I}{2\pi r} \\ L(r) &= \mu_0 \frac{r}{2} \end{aligned} \tag{18}$$

The four-vector gradient of the inductance is,

$$\partial_\alpha L(P) = \mu^\alpha \ \mathbf{Henrys\,/\,m} \tag{19}$$

Inductance then is also considered an effective potential that leads to a constant ratio of inductance per unit length, and a permeability vector μ^α. A function of the constant of permeability and the geometrical integral of the path.

Now consider that as constants the permeability and permittivity of free space are also Lorentz invariant. So what happens when the inductance and capacitance at the point P change due to the presence of polarizable matter nearby? The answer is obvious. Inductance and capacitance are strictly functions of the geometry. So in the same manner that Lorentz transformations preserve the constancy of the speed of light, they must also preserve the constancy of permeability and permittivity. Therefore natural coordinate transformations on the space-time manifold are functions of the polarizability represented by the inductance and capacitance at each point. Variation of these variables will necessitate transformations of the coordinates in order to preserve the Lorentz invariant constants.

Gauge Transformation

For the rest of this paper the properties of inductance and capacitance are collectively referred to as the *polarizability of the manifold at the point **P***. They are functions determined by the density and distribution of polarizable wave functions on the manifold. Transformations of the polarizability from point to point induce phase shifts in the propagator analogous to impedance transformations along a wave transmission line. On a transmission line, such transformations can lead to reflections and standing waves. This is equivalent to adding a background electromagnetic gauge potential to the existing gauge potential.

In what follows the reader must be careful to distinguish between the affects of a Lorentz transformation or coordinate transformation,. and that of a Gauge transformation. Both are discussed in order to illustrate the connection between Lorentz invariance and gauge invariance.

In a Minkowski space-time the gauge potentials of the Maxwell field provide the equations,

$$\begin{aligned} A_\alpha &= \left(-\phi, \vec{\mathbf{A}}\right) \\ F_{\alpha\beta} &= \partial_\alpha A_\beta - \partial_\beta A_\alpha \\ \partial_\beta F^{\alpha\beta} &= J^\alpha \end{aligned} \tag{20}$$

Where $F_{\alpha\beta}$ is the field strength and J^α is the current density at the sources. The polarizability added to empty space-time by the presence of matter changes the gauge potential through the interaction of the field with matter. The polarization matrix $M_{\alpha\beta}$, also known as the *Moments tensor* is added to the field strength.

$$\begin{aligned} A_\alpha &\to A_\alpha + B_\alpha \\ F_{\alpha\beta} &\to F_{\alpha\beta} + M_{\alpha\beta} \end{aligned} \tag{21}$$

The field B_α is taken to be that of a background potential relative to a test particle due to the presence of matter in its neighborhood. This is the effective potential resulting from the polarizability of the manifold. Lorentz transformations of the components of the Moments tensor can then be seen immediately as transformations in the values of inductance and capacitance.

Quantum mechanically we can impose the loose restriction that the expectation values of $M_{\alpha\beta}$ equal zero, and also that there are no nearby sources in free space. The expectation values of the field are then normalized to the vacuum field at that point.

$$\begin{aligned} \left\langle M_{\alpha\beta} \right\rangle_P &= 0 \\ \left\langle \partial_\beta M^{\alpha\beta} \right\rangle_P &= 0 \end{aligned} \tag{22}$$

The field $M_{\alpha\beta}$ then becomes simply the Zero Point electromagnetic field (ZPF). Since it has no contribution to Maxwell's equations, and leaves the expected field strength $F_{\alpha\beta}$ invariant under this transformation, it is essentially a *gauge transformation*.

Gauge transformations affect the phase of the wave function. Since the charge density J^α is held *gauge invariant*, then the addition of the gauge transformation changes the polarizability along an open path from the point $A \to B$.

$$\begin{aligned} \phi &\to \phi - \frac{\partial \chi}{\partial t} \\ \frac{q}{C(r)} &\to \frac{q}{C(r) + C'} \\ \vec{\mathbf{A}} &\to \vec{\mathbf{A}} + \vec{\nabla}\chi \\ L &\to \frac{1}{I}\left[\int_A^B \vec{\mathbf{A}} \bullet d\vec{s} + \left(\chi(B) - \chi(A)\right)\right] \end{aligned} \tag{23}$$

Note that the inductance and capacitance are both changed equally in such a way as to preserve the constant ratios of permeability, permittivity and the speed of light. This necessitates a transformation of the coordinates. In particular note that the spectral energy density ρ of the ZPF is Lorentz invariant [2]. Therefore there will not be any frequency dependence in the coordinate transformation induced by the gauge transformation. Therefore one would not expect there to be any chromatic dispersion, or colorful prismatic effects caused by the ZPF.

Invariance of Total Probability

The electromagnetic background can be thought of as the probability of an interaction between a test particle and the field in its immediate environment. The interaction can be defined as an exchange of some quantity of information in the form of energy, momentum , angular momentum, polarization or charge, mediated by real or virtual photons. The wave function of the test particle is that of equation (1) , a solution of Dirac's equation. The probability density for this particle is found by multiplying the probability amplitude by its *complex conjugate* probability wave,

$$\Psi = \psi^* \psi \qquad (24)$$

The total probability is,

$$\int \psi^* \psi \cdot dx = 1 \qquad (25)$$

The total probability normalized to 1 implies that the particle must be somewhere along x . Now if a Lorentz transformation is done on the coordinate x , then in the new coordinates x' the total probability is unaffected, meaning it is Lorentz invariant. *The probability density however is not Lorentz invariant and must change to preserve the total probability!*

$$\int \Psi' \cdot dx' = 1 \qquad (26)$$

Speculate for a moment that the conjugate wave function ψ^* is actually the *conjugate reflection* of the wave function ψ itself. There is some coefficient of reflection $R(\rho)$ that reverses the sign of the imaginary exponential and *may change the amplitude of the wave*, thereby changing the probability density. This function could for instance be determined by the spectral energy density ρ of the ZPF resulting from the interaction of the test particle with the matter and energy in the environment. Simply put, the greater the density of matter and energy the greater the probability of an interaction.

The conjugate wave is *space-time reversed.* Its phase advances in the opposite direction in both space and time relative to the wave function of the test particle. The interaction between the test particle and its conjugate reflection must depend on the relative refractive index as a function of the coordinates through which it is passing.

In the Probability Wave Dispersion Interpretation the wave function is reflected by interactions with the background field as the wave propagates along a path. In this manner gradients in the refractive index cause reflection *and absorption* which change the probability density and lead to transformations of the coordinates. This is identical to what was described above when the coordinates were intentionally assumed to have been Lorentz transformed.

$$\int R(\rho)\Psi dx' = 1 \tag{27}$$

There is no reason to suppose that the coefficient of reflection in each direction, or the spectral energy density of the potential B^{α} are constants at all coordinates. In fact one needs to suppose that they would depend on the density of matter and energy generating the background field $M_{\alpha\beta}$ at the point P. The gauge transformation is the gradient of a scalar function, and that gradient corresponds to the gradient in the probability of an interaction. Thus by means of a variable index of refraction resulting from the absorption and emission of photons the probability density is transformed into a function of the local density of matter and energy, *and so are the coordinates!*

Deriving the Metric

The contravariant four-vector for the relativistic energy and momentum of a free particle as a wave solution of the Dirac equation is,

$$p^{\alpha} = \left(\frac{E}{\mathbf{c}}, \vec{\mathbf{p}}\right) \tag{28}$$

The coordinate four-vector is,

$$x^{\alpha} = \left(\frac{x^{0}}{\mathbf{c}}, \vec{\mathbf{x}}\right) \tag{29}$$

If at each point we define a space-time manifold by its events and its shape as determined by its polarizability as hypothesized, then the gradient derivatives of the polarizability with respect to the parameter λ are tangent to the manifold at each point. The derivatives taken in Cartesian coordinates describe the permeability and permittivity at the point $\boldsymbol{P}$ as a pair of four-vectors tangent to the manifold. The gradients of capacitance and inductance. The speed of light, like permeability and permittivity is independent of the inertial coordinate system in which it is measured. It is determined by the inner product of these two vectors.

$$\begin{gathered} \mu^{\alpha}\eta_{\alpha\beta}\varepsilon^{\beta} = \frac{1}{\mathbf{c}^{2}} \\ \varepsilon^{\alpha}\eta_{\alpha\beta}\mu^{\beta} = \mathbf{c}^{2} \end{gathered} \tag{30}$$

By decomposing these two four-vectors into two unit four-vectors a^{α} and b^{α}, and normalizing the invariant, we have the inner product,

$$a^{\alpha} g_{\alpha\beta} b^{\beta} = 1 \tag{31}$$

which determines the components of the Refractive metric tensor [1].

$$g_{\alpha\beta} = \begin{bmatrix} \left(\frac{\mathbf{c}}{\mathbf{c}_{00}}\right)^2 & 0 & 0 & 0 \\ 0 & -\left(\frac{\mathbf{c}}{\mathbf{c}_{11}}\right)^2 & 0 & 0 \\ 0 & 0 & -\left(\frac{\mathbf{c}}{\mathbf{c}_{22}}\right)^2 & 0 \\ 0 & 0 & 0 & -\left(\frac{\mathbf{c}}{\mathbf{c}_{33}}\right)^2 \end{bmatrix} \tag{32}$$

To derive these components the field strength is broken into the source field and the polarization. Then solved for the components which are added to the identity tensor.

$$\begin{gathered} F^{\alpha\beta} = H^{\alpha\beta} + M^{\alpha\beta} \\ \left[F^{\alpha\beta}\right]\cdot\left[H^{\alpha\beta}\right]^{-1} = I^{\alpha\beta} + \left[M^{\alpha\beta}\right]\cdot\left[H^{\alpha\beta}\right]^{-1} \end{gathered} \tag{33}$$

These are the equations that determine the refractive index at each set of coordinates.

The Dispersion Interpretation of Inertia

The refractive metric is applied to the momentum four-vector to find the invariant length of that vector,

$$p^{\alpha} g_{\alpha\beta} p^{\beta} = (\mathrm{m}\mathbf{c})^2 \tag{34}$$

which is the invariant rest mass **m**. This equation is written out in component form in Cartesian coordinates as,

$$(\mathrm{m}\mathbf{c})^2 = \left(\frac{E}{\mathbf{c}_{00}}\right)^2 - \mathbf{c}^2\left[\left(\frac{\mathrm{p}_1}{\mathbf{c}_{11}}\right)^2 + \left(\frac{\mathrm{p}_2}{\mathbf{c}_{22}}\right)^2 + \left(\frac{\mathrm{p}_3}{\mathbf{c}_{33}}\right)^2\right] \tag{35}$$

and the total energy written in terms of the metric components is,

$$E = (\mathbf{c}_{00}\mathbf{c})\sqrt{\left(\frac{\mathrm{p}_1}{\mathbf{c}_{11}}\right)^2 + \left(\frac{\mathrm{p}_2}{\mathbf{c}_{22}}\right)^2 + \left(\frac{\mathrm{p}_3}{\mathbf{c}_{33}}\right)^2 + \mathrm{m}^2} \tag{36}$$

This is a relativistic energy equation for a probability wave solution on a polarizable space-time manifold. From the above equation Planck's constant can be factored out so that it can be written as a wave dispersion relationship,

$$\frac{\omega^2}{(\mathbf{c}_{00})^2\left[\left(\frac{\mathrm{k}_1}{\mathbf{c}_{11}}\right)^2 + \left(\frac{\mathrm{k}_2}{\mathbf{c}_{22}}\right)^2 + \left(\frac{\mathrm{k}_3}{\mathbf{c}_{33}}\right)^2 + \mathrm{m}^2\right]} = \mathbf{c}^2 \tag{37}$$

where the effective wavelength Λ is given as a function of the wave number k, $\Lambda = \frac{2\pi}{k}$ and,

$$k = \mathbf{c}_{00}\sqrt{\left(\frac{\mathrm{k}_1}{\mathbf{c}_{11}}\right)^2 + \left(\frac{\mathrm{k}_2}{\mathbf{c}_{22}}\right)^2 + \left(\frac{\mathrm{k}_3}{\mathbf{c}_{33}}\right)^2 + \mathrm{m}^2} \tag{38}$$

These are the equations that govern the dispersion of the probability waves in a polarizable Quantum vacuum such as the Dirac sea [2]. Einstein's principle of relativity is *enforced* by these two equations.

For example. Given a real force that acts on the particle of mass **m**., such as in a particle accelerator. We observe that the rest mass remains invariant *and* constant, since it is an intrinsic property of the particle. However the total energy and momentum of the particle change as a function of the work done to it. They determine the effective wavelength of the particle. As the total energy and momentum increase the effective wavelength decreases. This is interpreted as the contraction of length along the direction of travel caused by the applied force. It is a direct result of the relativistic dispersion relationship of the wave function. The relative contraction corresponds exactly to the amount of energy input by the work done to accelerate the particle from the momentum **p** to **p+dp**. Thus it accounts precisely for all of the inertia "felt" as the reaction force, to the applied force. It is interpreted such that the work done to accelerate an object physically contracts the wave functions that comprise the object. Therefore there is *resistance to acceleration,* which is then quantified as the *rest mass* of the object in the classical sense, $\vec{\mathbf{F}} = \mathrm{m}\vec{\mathbf{a}}$.

The Dispersion Interpretation of Space-time Curvature

Now consider the derivatives of the metric tensor. These are known as the Christoffel field. Using comma "," notation for the partial derivatives with respect to the coordinates, the Christoffel symbols are the functions,

$$\Gamma^{\gamma}_{\alpha\beta} = \frac{1}{2} g^{\gamma\sigma}\left(g_{\sigma\alpha,\beta} + g_{\sigma\beta,\alpha} - g_{\alpha\beta,\sigma}\right) \tag{39}$$

These derivatives are typically used to show the potentials of fictitious forces such as the Coriolis force. Using a Minkowski metric in Cartesian coordinates these derivatives vanish. However using the Refractive metric, *the derivatives will only vanish if the permeability and permittivity four-vectors are constant!* Otherwise these derivatives determine the geodesic motion of test particles that are solutions to the Dirac equation, on the polarizable space-time manifold. They can also be used to compute the curvature tensor, in terms of which the vacuum field equations of General Relativity exist.

Using the definition of the four-velocity,

$$u^{\alpha} = \frac{\mathrm{d}}{\mathrm{ds}} x^{\alpha} \tag{40}$$

where the invariant space-time interval is,

$$(\mathrm{ds})^2 = g_{\alpha\beta}\,\mathrm{d}x^\alpha \mathrm{d}x^\beta \qquad (41)$$

If the effect of the polarizability is small, then a solution to the Dirac equation is a plane wave solution of the form,

$$\psi_\pm(x,p) = e^{\pm i\left(p^\alpha g_{\alpha\beta} x^\alpha\right)}\zeta^\ell(p) \qquad (42)$$

where $\psi_\pm$ are positive and negative solutions, and ζ^ℓ represents the four Spinor solutions of the angular momentum. For this wave function, free fall on the manifold is constrained by the geodesic equation of motion,

$$\mathrm{d}u^\gamma + \Gamma^\gamma_{\alpha\beta} u^\alpha \mathrm{d}x^\beta = 0 \qquad (43)$$

One can always specify coordinates such that at any given point, for an instant of time the Christoffel symbols are zero. In other words, space-time is "flat" locally. These are referred to as *geodesic coordinates* [3].

This geodesic motion is analogous to a simple harmonic oscillator consisting of a parallel connected inductor and capacitor, ideally with zero resistance and a constant quantity of electric charge oscillating between them. The frequency of oscillation is determined by the values of inductance L and capacitance C in the circuit. The energy stored by the oscillator is also determined by these values. As they increase the energy stored decreases in inverse proportion. The dispersion of a wave packet is then analogous to the discharge of energy by increasing the values of the inductor and capacitor in the electronic oscillator circuit, while keeping the charge contained therein constant

The Schwarzschild Solution

To show that the Refractive metric is a function of the mass and energy of objects in the environment we consider the Refractive metric components in spherical polar coordinates. It is assumed there is a central mass at the origin of the coordinate system. In terms of the refractive index $\mathrm{n}(r)$ the metric is,

$$g_{\alpha\beta} = \begin{bmatrix} \left(\frac{1}{\mathrm{n}(r)}\right)^2 & 0 & 0 & 0 \\ 0 & -(\mathrm{n}(r))^2 & 0 & 0 \\ 0 & 0 & -r^2 & 0 \\ 0 & 0 & 0 & -r^2\sin^2\theta \end{bmatrix} \qquad (44)$$

It is interpreted such that the refractive index $\mathrm{n} \geq 1$, therefore the quantity g_{00} can be expanded into a simple function of r and k, where r is the distance from the origin of the coordinates.

A wave function has a characteristic wavelength, and its intrinsic dipole moment is proportional to the distance between opposite amplitudes of the wave. Therefore it is proportional to half the effective wavelength $\frac{\Lambda}{2}$, or inversely proportional to $2 \cdot k$. The simplest function of the proper form can be *reverse engineered* to fit the known solution,

$$g_{00} = \left(\frac{1}{\mathrm{n}(r)}\right)^2 = 1 - \frac{2k}{r} \tag{45}$$

In the case of a motionless central mass and a negligible test particle the momentum is zero, so $k \propto \mathrm{m}$.

When referring to the polarizability of the wave functions at the point ***P***, this is actually in the spirit of *Mach's* principle. A wave function in free space has no boundary, so there is non-zero probability for any wave function to affect the polarizability at any point as a function of the coordinates. This is consistent with the Transactional Interpretation of Quantum Mechanics. The greater the number of particles, the greater the probability of a transaction. If the central object consists of **N** particles of mass **m**, then this leads directly to the Schwarzschild solution of the vacuum equation of General Relativity. Written in the usual form as a function of r, with $\mathbf{c} = 1$, the total mass is given by the inverse of the effective wavelength,

$$\mathrm{M} = \sum_{\mathrm{N}} k = \sum_{\mathrm{N}} \frac{2\pi}{\Lambda} \tag{46}$$

Note that in geometrical units mass always has units of inverse length. Therefore we have found the Schwarzschild metric can be expressed as a function of the polarizability in the region of the point ***P***,

$$g_{\alpha\beta} = \begin{bmatrix} 1-\frac{2\mathrm{M}}{r} & 0 & 0 & 0 \\ 0 & \frac{-1}{1-\frac{2\mathrm{M}}{r}} & 0 & 0 \\ 0 & 0 & -r^2 & 0 \\ 0 & 0 & 0 & -r^2\sin^2\theta \end{bmatrix} \tag{47}$$

Thus showing that the properties of mass, inertia and space-time curvature are just different aspects of the dispersion of probability waves.

Dispersion and Geometry

Dispersion diagrams are typically used in the theory of waves propagating along transmission lines, or through wave guides [4]. The local distribution of events in space-time determines the shape and size of these wave guides. The dispersion diagram illustrates the relationship between frequency and wavelength.

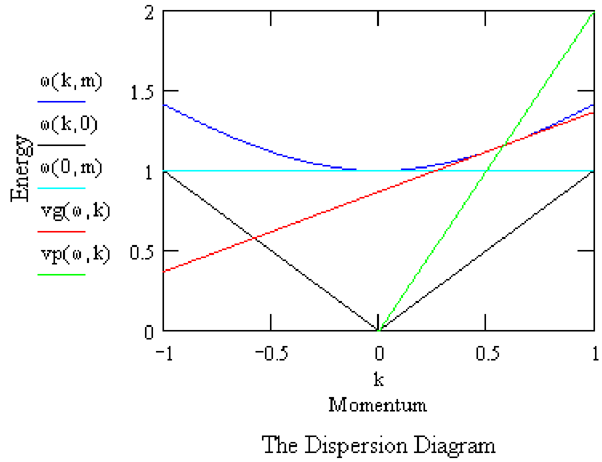

FIG. 1

The equation plotted here in blue for a mass of 1 and a range of momentum from $-1 \le \mathrm{k} \le 1$, is the total energy, which is the hyperbola,

$$\omega(\mathrm{k}, m) = \sqrt{\mathrm{k}^2 + m^2} \tag{48}$$

Graphs for conditions of zero mass photons $\omega(\mathrm{k},0)$, and rest energy $\omega(0,1)$ are also shown as diagonal and horizontal straight lines respectively.

The interesting plots are the group velocity $\mathrm{vg(k)}$, and the phase velocity $\mathrm{vp(k)}$. The group velocity, shown in red is tangent to the hyperbola, and is defined as the derivative,

$$\mathrm{v}g(\mathrm{k}) = \frac{\mathrm{d}\omega}{\mathrm{dk}} \tag{49}$$

The phase velocity shown in green goes through the origin and intersects the hyperbola at the point tangent to the group velocity. It is defined by the values of energy and momentum at that point,

$$\mathrm{vp(k)} = \frac{\omega}{\mathrm{k}} \tag{50}$$

For a null geodesic line of a ray of light, the group and phase velocities are equal.

$$\mathrm{vg(k)} = \mathrm{vp(k)} = \mathbf{c} = 1 \tag{51}$$

Noting that the intersection of the group and phase velocities can be transposed to the origin of a Cartesian coordinate system. The result is a Minkowski space-time diagram.

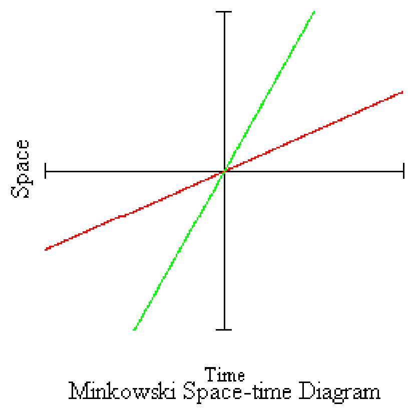

FIG. 2

The group velocity is then,

$$vg(x,t) = \frac{dx}{dt} \quad (52)$$

Therefore this diagram shows that the Dispersion interpretation and Minkowski's Geometrical interpretation of Relativity are equivalent.

Conclusion

Rather than a universal constant, the speed of light is shown to be an invariant quantity determined by the tangent vector fields of the capacitance and inductance gradients. The variation of these vector components from point to point determines the geodesic motion of test particles resulting from the dispersion of their probability waves. From the hypothesis that the shape of the space-time manifold is determined by the polarizability of wave functions, a new interpretation of Relativity is brought about and the properties of mass, inertia and space-time curvature are shown to be merely different aspects of wave dispersion. The relationship between the Refractive metric and the Schwarzschild metric, as well as the equivalence of the dispersion diagram to the Minkowski space-time diagram shows that the theory of Quantum Electrodynamics and the Dirac equation lead logically, and consistently to a viable theory of Quantum Gravity. Other evidence not shown that supports the equivalence of these two interpretations is the gravitational red shift and gravitational lensing, both of which can easily be derived from the Refractive metric components.

References:

1. Bjorn Felsager, *"Geometry, Particles and Fields"* 1998 Springer-Verlag New York, Inc. The derivation closely follows the derivation in Ch. 6, pgs. 296-297.
2. P. W. Milonni, *"The Quantum Vacuum, An Introduction to Quantum Electrodynamics"*, 1994 Academic Press, Ch. 1, 2, 10.
3. Alan C. Tribble *"Princeton Guide to Advanced Physics"*, 1996 Princeton University Press, pg. 181.
4. R. H. Rumsey, "Diffraction Informatics" UCSD, 1988.